## Comparing the stability of tribenzo[b,n,pqr]perylene and tribenzo[b,k,pqr]perylene

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**Abstract** Two isomeric benzenoid hydrocarbons – tribenzo[b,n,pqr]perylene and tribenzo[b,k,pqr]perylene played a crucial role in the formulation of the Clar aromatic sextet theory. The basic assumption of this theory is that tribenzo[b,n,pqr]perylene is more stable than tribenzo [b,k,pqr] perylene because the former has five, whereas the latter only four aromatic sextets. We now approach this stability problem from a different direction. By means of a recently developed molecular-orbital-based method it is possible to estimate the energy effects of individual cycles, as well as pairs, triplets, etc. of cycles in polycyclic conjugated molecules. From these energy-effects one can better understand which structural details are responsible for the thermodynamic stability of the underlying molecule. In particular, it is possible to rationalize (in a quantitative manner) the causes of differences in the thermodynamic stability of isomers. Our analysis corroborates the conclusion of Clar theory, but points out a number of hitherto overlooked structure-stability connections.

**Keywords** *Clar* theory; Cyclic conjugation; Energy effect of cyclic conjugation; Tribenzoperylene.

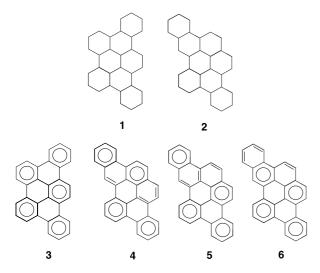
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## Introduction

The comparison of two isomeric tribenzoperylenes (1 and 2, see Fig. 1) played a crucial role in the discovery of the *Clar aromatic sextet theory*.

In the 1950s Eric Clar together with his student Maximilian Zander [1] synthesized two isomeric tribenzoperylenes and found that tribenzo[b,k,pqr]perylene (2) is significantly more reactive than tribenzo[b,n,pqr]perylene (1). Clar connected these experimental observations with the fact that the 30  $\pi$ -electrons of 1 can be divided into five groups of six, and each such group (referred to as an "aromatic sextet") can be formally located in one of the five mutually disjoint six-membered rings of 1. In the case of 2 no such partitioning of  $\pi$ -electrons is possible: of the  $30\pi$ -electrons of 2 only 24 can be grouped into aromatic sextets whereas the remaining 6 must belong to ordinary double bonds. When the aromatic sextets are represented by circles, we arrive at the diagrams that nowadays are known as Clar aromatic sextet formulas, see Fig. 1. In Clar's opinion [1, 2], the isomer 1 is more stable than 2 because the Clar formula of 1 has five, whereas any of the Clar formulas of 2 has only four aromatic sextets.

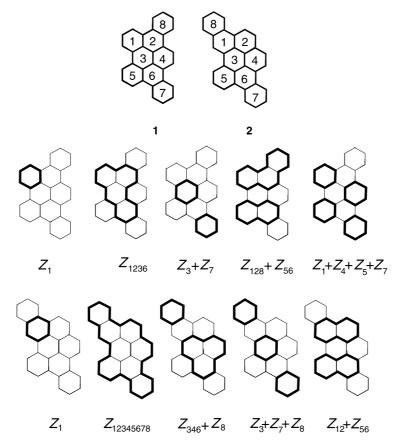
Generalizing this way of looking at the electronic structure of benzenoid hydrocarbons, *Clar* arrived at what nowadays is called the *Clar aromatic sextet theory* [2, 3]. Eventually, this theory became one of the standard approaches by means of which the



**Fig. 1** Tribenzo[b, n, pqr]perylene (1), tribenzo[b, k, pqr]perylene (2), and their *Clar* aromatic sextet formulas. Whereas 1 has a unique sextet formula with five aromatic sextets (3), its isomer 2 is represented by three such formulas (4, 5, 6), each with 4 aromatic sextets

peculiar physico-chemical properties of benzenoid molecules can be rationalized [2–4]. Research along these lines is currently very active, see the reviews [4, 5], the papers [6–8], and the references cited therein.

A critical-minded chemist may argue that *Clar*'s reasoning [1, 2] is no *explanation* whatsoever why tribenzo[b,n,pqr]perylene is more stable than tribenzo[b,k,pqr]perylene. Namely, there is no physical argument (especially, no such argument coming from quantum theory) that would relate the number of aromatic sextets with the stability (especially, thermodynamic stability) of a benzenoid molecule. The only reason why we may *believe* in *Clar* theory is that there exist numerous examples that the stability of isomeric benzenoid compounds increases with the number of their aromatic sextets. There, however, exist exceptions of this rule [8–10].



**Fig. 2** The numbering of the six-membered rings of the tribenzoperylenes **1** and **2**, and examples showing the way in which their cycles, pairs of disjoint cycles, triplets of mutually disjoint cycles, *etc.* are denoted. Tribenzo[*b,n,pqr*]perylene (**1**) has a total of 109 cycles, 106 pairs of disjoint cycles, as well as 34 triples, 5 quartets, and 1 quintet of mutually disjoint cycles. Tribenzo[*b,k,pqr*]perylene (**2**) has a total of 104 cycles, 114 pairs of disjoint cycles, as well as 32 triples and 5 quartets of mutually disjoint cycles

In this paper we show how the stability order of the tribenzoperylene isomers could be explained by using a completely different way of reasoning, namely by comparing the energy-effects of their various cycles.

There are several possible ways to quantify cyclic conjugation of polycyclic conjugated molecules. Details of these approaches can be found in the books [2, 3] and reviews [4, 5]. Another, conceptually different approach is the (molecular-orbitalbased) calculation of the energy-effect that results from conjugation in a particular cycle, or pairs of cycles, or triplets of cycles, etc. A pioneering work along these lines was published already in the 1960s [11], but the complete theory could be formulated only after appropriate graph-theoretical methods were developed. The energy-effects of individual cycles in polycyclic conjugated molecules were first calculated by one of the present authors [12, 13], and independently (in a somewhat different manner) by Aihara [14]. Details of this theory of cyclic conjugation are found in the review [15] and the recent paper [16].

Let G be the molecular graph [17], representing a polycyclic conjugated molecule, and Z a cycle contained in it. Then the energy-effect resulting from the cyclic interaction of the  $\pi$ -electrons along Z is expressed as

$$ef(Z) = ef(Z \mid G)$$

$$= \frac{2}{\pi} \int_0^\infty \ln \left| \frac{\phi(G, ix) + 2\phi(G - Z, ix)}{\phi(G, ix)} \right| dx \quad (1)$$

where  $\phi(G, x)$  is the characteristic polynomial of G, and  $\phi(G - Z, x)$  is the characteristic polynomial of the subgraph G - Z, obtained by deleting the cycle Z from G. Further,  $i = \sqrt{-1}$ .

The analogous energy-effect of two disjoint cycles  $Z_1$  and  $Z_2$  is

$$ef(Z_{1},Z_{2}) = ef(Z_{1},Z_{2} | G)$$

$$= \frac{2}{\pi} \int_{0}^{\infty} \ln \left| \frac{\phi(G,ix) - 4\phi(G - Z_{1} - Z_{2},ix)}{\phi(G,ix)} \right| dx$$

of three mutually disjoint cycles  $Z_1$ ,  $Z_2$ , and  $Z_3$  is

$$\begin{aligned} ef(Z_1, Z_2, Z_3) &= ef(Z_1, Z_2, Z_3 \mid G) \\ &= \frac{2}{\pi} \int_0^\infty \ln \left| \frac{\phi(G, ix) + 8\phi(G - Z_1 - Z_2 - Z_3, ix)}{\phi(G, ix)} \right| dx \end{aligned}$$

and in the general case,

$$ef(Z_{1},...,Z_{n}) = ef(Z_{1},...,Z_{n}|G)$$

$$= \frac{2}{\pi} \int_{0}^{\infty} \ln \left| \frac{\phi(G,ix) - (-2)^{n} \phi(G - Z_{1} - \dots - Z_{n},ix)}{\phi(G,ix)} \right| dx.$$
(2)

If the cycles are not disjoint, then their collective energy effect is zero. For more details on the quantity *ef* see Refs. [15, 16] and the references cited therein.

The above-specified energy-effects are expressed in the units of the HMO carbon-carbon resonance integral  $\beta$ , whose value is negative. Therefore, if ef > 0, then the respective interaction of  $\pi$ -electrons causes thermodynamic stabilization. Otherwise, ef < 0 implies thermodynamic destabilization.

## Results and discussion

Cycles in tribenzoperylenes

In Fig. 2 is indicated the labelling of the rings of the two tribenzoperylenes considered, and examples are provided clarifying the way in which their

**Table 1** The energy-effects (in  $\beta$ -units) of the six- and tenmembered cycles of tribenzoperylenes **1** and **2**, cf. Fig. 1. The labelling of the cycles is indicated in Fig. 2

Cycle	Size	1	2
$\overline{Z_1}$	6	0.1462	0.0553
$Z_2$	6	0.0264	0.0757
$Z_3$	6	0.0255	0.0231
$Z_4$	6	0.1124	0.0901
$Z_5$	6	0.1462	0.1496
$Z_6$	6	0.0264	0.0285
$Z_7$	6	0.1886	0.1840
$Z_8$	6	0.1886	0.1386
$Z_{12}$	10	0.0096	0.0113
$Z_{13}$	10	0.0083	0.0033
$Z_{18}$	10	_	0.0311
$Z_{23}$	10	0.0013	0.0034
$Z_{24}$	10	0.0066	0.0251
$Z_{28}$	10	0.0111	_
$Z_{34}$	10	0.0070	0.0044
$Z_{35}$	10	0.0083	0.0064
$Z_{36}$	10	0.0013	0.0013
$Z_{46}$	10	0.0066	0.0054
$Z_{56}$	10	0.0096	0.0111
$Z_{67}$	10	0.0111	0.0128

I. Gutman, S. Stanković

**Table 2** Same data as in Table 1 for pairs of disjoint six- and ten-membered cycles

Cycle	Size	1	2	Cycle	Size	1	2
$\overline{Z_1 + Z_4}$	6+6	0.0324	0.0111	$Z_6 + Z_{12}$	6+10	0.0006	0.0007
$Z_1 + Z_5$	6 + 6	0.0339	0.0119	$Z_6 + Z_{18}$	6 + 10	_	0.0008
$Z_1 + Z_6$	6 + 6	0.0031	0.0012	$Z_6 + Z_{28}$	6 + 10	0.0007	_
$Z_1 + Z_7$	6 + 6	0.0337	0.0118	$Z_7 + Z_{12}$	6 + 10	0.0022	0.0026
$Z_1 + Z_8$	6 + 6	0.0433	_	$Z_7 + Z_{13}$	6 + 10	0.0022	0.0008
$Z_2 + Z_5$	6 + 6	0.0031	0.0118	$Z_7 + Z_{18}$	6 + 10	_	0.0090
$Z_2 + Z_6$	6 + 6	0.0012	0.0040	$Z_7 + Z_{23}$	6 + 10	0.0002	0.0005
$Z_2 + Z_7$	6 + 6	0.0037	0.0138	$Z_7 + Z_{24}$	6 + 10	0.0022	0.0093
$Z_2 + Z_8$	6 + 6	_	0.0196	$Z_7 + Z_{28}$	6 + 10	0.0024	_
$Z_3 + Z_7$	6 + 6	0.0037	0.0028	$Z_7 + Z_{34}$	6 + 10	0.0022	0.0012
$Z_3 + Z_8$	6 + 6	0.0037	0.0017	$Z_7 + Z_{35}$	6 + 10	0.0024	0.0016
$Z_4 + Z_5$	6 + 6	0.0324	0.0249	$Z_8 + Z_{13}$	6 + 10	0.0024	_
$Z_4 + Z_7$	6 + 6	0.0337	0.0261	$Z_8 + Z_{23}$	6 + 10	_	0.0005
$Z_4 + Z_8$	6 + 6	0.0337	0.0131	$Z_8 + Z_{24}$	6 + 10	_	0.0090
$Z_5 + Z_7$	6 + 6	0.0433	0.0446	$Z_8 + Z_{34}$	6 + 10	0.0022	0.0004
$Z_5 + Z_8$	6 + 6	0.0337	0.0261	$Z_8 + Z_{35}$	6 + 10	0.0022	0.0007
$Z_6 + Z_8$	6 + 6	0.0037	0.0041	$Z_8 + Z_{36}$	6 + 10	0.0002	0.0001
$Z_7 + Z_8$	6 + 6	0.0433	0.0287	$Z_8 + Z_{46}$	6 + 10	0.0022	0.0009
$Z_1 + Z_{46}$	6 + 10	0.0020	0.0007	$Z_8 + Z_{56}$	6 + 10	0.0022	0.0025
$Z_1 + Z_{56}$	6 + 10	0.0021	0.0008	$Z_8 + Z_{67}$	6 + 10	0.0024	0.0028
$Z_1 + Z_{67}$	6 + 10	0.0021	0.0008	$Z_{12} + Z_{56}$	10 + 10	0.0004	0.0005
$Z_2 + Z_{56}$	6 + 10	0.0006	0.0024	$Z_{12} + Z_{67}$	10 + 10	0.0004	0.0005
$Z_2 + Z_{67}$	6 + 10	0.0007	0.0028	$Z_{18} + Z_{46}$	10 + 10	_	0.0005
$Z_4 + Z_{18}$	6 + 10	_	0.0086	$Z_{18} + Z_{56}$	10 + 10	_	0.0006
$Z_5 + Z_{12}$	6 + 10	0.0021	0.0026	$Z_{18} + Z_{67}$	10 + 10	_	0.0006
$Z_5 + Z_{18}$	6 + 10	_	0.0091	$Z_{28} + Z_{56}$	10 + 10	0.0004	_
$Z_5 + Z_{24}$	6 + 10	0.0020	0.0087	$Z_{28} + Z_{67}$	10 + 10	0.0005	_
$Z_5 + Z_{28}$	6 + 10	0.0021	_				

cycles, as well as pairs, triples, *etc*. of disjoint cycles are denoted. For instance, in **1**,  $Z_{1236}$  denotes the 17-membered cycle embracing the rings 1, 2, 3, and 6. The 26-membered cycle, embracing all the eight rings of **8**, is denoted by  $Z_{12345678}$ ; thus  $Z_{12345678}$  is the perimeter of **2**. By  $Z_{28} + Z_{56}$  (not shown in Fig. 2) we would denote a pair of two disjoint 10-membered cycles of **1**, embracing the rings 2, 5, 6, and 8. On the other hand, the same four rings of **2** would induce a triple of mutually disjoint cycles  $-Z_2 + Z_8 + Z_{56}$ , consisting of two 6-membered and one 10-membered cycle.

The examination of cyclic conjugation of a large number of benzenoid hydrocarbons [15] showed that in the majority of cases, the energy-effect rapidly decreases when the size of the cycle increases. This means that the greatest energy effects have the six-membered cycles, the energy effects of 10-membered cycles are smaller, *etc.* For this reason, as well as because of the very large number of cycles in the tribenzoperylenes (*cf.* Fig. 2), in what

follows we consider and discuss only the *ef*-values of 6- and 10-membered cycles, and of their disjoint combinations.<sup>1</sup>

At this point we mention that in benzenoid systems, all cycles of size 4k + 2,  $k = 1, 2, 3, \ldots$ , have a stabilizing energy-effect (positive ef) [15, 18]. The cycles of size  $4k, k = 3, 4, 5, \ldots$ , have a destabilizing energy-effect (negative ef) [15, 18]. The energy-effects of the (4k)-membered cycles in tribenzo-perylenes are very small, and therefore their neglect is fully justified. For instance, for the 12-membered cycles  $Z_{123}$  and  $Z_{346}$  we have  $ef(Z_{123} | \mathbf{1}) = -0.0005$ ,  $ef(Z_{123} | \mathbf{2}) = -0.0006$ ,  $ef(Z_{346} | \mathbf{1}) = -0.0004$ , and  $ef(Z_{346} | \mathbf{2}) = -0.0003$ , values that should be compared with those in Tables 1 and 2. The energy-effects of (4k)-membered cycles whose size is greater than 12 are practically equal to zero.

<sup>&</sup>lt;sup>1</sup> The *ef*-values of all cycles and disjoint combinations thereof of **1** and **2** are available from the authors, upon request.

**Table 3** Same data as in Table 1 for triplets of mutually disjoint six- and ten-membered cycles

Cycle	Size	1	2
$\overline{Z_1 + Z_4 + Z_5}$	6+6+6	0.0127	0.0043
$Z_1 + Z_4 + Z_7$	6 + 6 + 6	0.0127	0.0043
$Z_1 + Z_4 + Z_8$	6 + 6 + 6	0.0130	_
$Z_1 + Z_5 + Z_7$	6 + 6 + 6	0.0130	0.0045
$Z_1 + Z_5 + Z_8$	6 + 6 + 6	0.0130	_
$Z_1 + Z_6 + Z_8$	6 + 6 + 6	0.0010	_
$Z_1 + Z_7 + Z_8$	6 + 6 + 6	0.0130	_
$Z_2 + Z_5 + Z_7$	6 + 6 + 6	0.0010	0.0045
$Z_2 + Z_5 + Z_8$	6 + 6 + 6	_	0.0044
$Z_2 + Z_6 + Z_8$	6 + 6 + 6	_	0.0013
$Z_2 + Z_7 + Z_8$	6 + 6 + 6	_	0.0049
$Z_3 + Z_7 + Z_8$	6 + 6 + 6	0.0010	0.0003
$Z_4 + Z_5 + Z_7$	6 + 6 + 6	0.0130	0.0100
$Z_4 + Z_5 + Z_8$	6 + 6 + 6	0.0127	0.0047
$Z_4 + Z_7 + Z_8$	6 + 6 + 6	0.0130	0.0048
$Z_5 + Z_7 + Z_8$	6 + 6 + 6	0.0130	0.0100
$Z_1 + Z_8 + Z_{46}$	6 + 6 + 10	0.0008	_
$Z_1 + Z_8 + Z_{56}$	6 + 6 + 10	0.0008	_
$Z_1 + Z_8 + Z_{67}$	6 + 6 + 10	0.0008	_
$Z_2 + Z_8 + Z_{56}$	6 + 6 + 10	_	0.0010
$Z_2 + Z_8 + Z_{67}$	6 + 6 + 10	_	0.0011
$Z_4 + Z_5 + Z_{18}$	6 + 6 + 10	_	0.0036
$Z_4 + Z_7 + Z_{18}$	6 + 6 + 10	_	0.0036
$Z_5 + Z_7 + Z_{12}$	6 + 6 + 10	0.0008	0.0010
$Z_5 + Z_7 + Z_{18}$	6 + 6 + 10	_	0.0038
$Z_5 + Z_7 + Z_{24}$	6 + 6 + 10	_	0.0038
$Z_5 + Z_7 + Z_{28}$	6 + 6 + 10	0.0008	_
$Z_5 + Z_8 + Z_{24}$	6 + 6 + 10	_	0.0036
$Z_7 + Z_8 + Z_{13}$	6 + 6 + 10	0.0008	_
$Z_7 + Z_8 + Z_{23}$	6 + 6 + 10	_	0.0001
$Z_7 + Z_8 + Z_{24}$	6 + 6 + 10	_	0.0038
$Z_7 + Z_8 + Z_{34}$	6 + 6 + 10	0.0008	0.0001
$Z_7 + Z_8 + Z_{35}$	6 + 6 + 10	0.0008	0.0002

**Table 4** Same data as in Table 1 for quartets of mutually disjoint six- and ten-membered cycles. Also the energy-effect of the five mutually disjoint six-membered cycles of **1** is included

Cycle	Size	1	2
$\overline{Z_1 + Z_4 + Z_5 + Z_7}$	6+6+6+6	0.0055	0.0019
$Z_1 + Z_4 + Z_5 + Z_8$	6+6+6+6	0.0055	_
$Z_1 + Z_4 + Z_7 + Z_8$	6+6+6+6	0.0055	_
$Z_1 + Z_5 + Z_7 + Z_8$	6+6+6+6	0.0055	_
$Z_2 + Z_5 + Z_7 + Z_8$	6+6+6+6	_	0.0019
$Z_4 + Z_5 + Z_7 + Z_8$	6+6+6+6	0.0055	0.0020
$Z_4 + Z_5 + Z_7 + Z_{18}$	6+6+6+10	_	0.0016
$Z_5 + Z_7 + Z_8 + Z_{24}$	6+6+6+10	_	0.0016
$Z_1 + Z_4 + Z_5 +$	6+6+6+	0.0025	_
$Z_7 + Z_8$	6 + 6		

Cyclic conjugation in tribenzoperylenes

Using Eqs. (1) and (2) we have computed all cyclic-conjugation energy-effects in the tribenzoperylenes 1 and 2. The respective results, pertaining to their six- and ten-membered cycles, are given in Tables 1–4.

By inspection of the Tables 1–4 we first conclude that the cyclic conjugation energy-effects of single six-membered and single ten-membered cycles are far the strongest. Some energy-effects of pairs of six-membered cycles are also not negligible (Table 2), but all other *ef*-values are two or more orders of magnitude smaller than those of single six-membered cycles. (Equally small, or even smaller, are the *ef*-values pertaining to cycles greater than 10, not shown in Tables 1–4.)

Therefore, although in the tribenzoperylenes 1 and 2 there are very many different modes of cyclic conjugation, the main cause of the difference in their thermodynamic stability are the energy-effects of their six-membered and (to a lesser extent) ten-membered cycles.

From Figs. 1 and 2 we see that in the isomers **1** and **2** one may directly compare the energy-effects resulting from cyclic conjugation in the rings 3, 4, 5, 6, and 7. Some of these are nearly equal, in particular  $ef(Z_3)$ ,  $ef(Z_5)$ ,  $ef(Z_6)$ ,  $ef(Z_7)$ ,  $ef(Z_{36})$ , and  $ef(Z_5 + Z_7)$ . Some energy-effects slightly favor the isomer **1**, in particular,  $ef(Z_{34})$ ,  $ef(Z_{35})$ ,  $ef(Z_{46})$ ,  $ef(Z_3 + Z_7)$ ,  $ef(Z_4 + Z_5)$ , and  $ef(Z_4 + Z_5)$ , whereas a few slightly favor the isomer **2**, in particular  $ef(Z_{56})$  and  $ef(Z_{67})$ . However, only one of these energy-effects, namely of  $Z_4$ , is much stronger in **1** than in **2**. Indeed,

$$ef(Z_4 | \mathbf{1}) = 0.112$$
  
 $ef(Z_4 | \mathbf{2}) = 0.090$ .

The effects of cyclic conjugation resulting from the rings 1, 2, and 8 cannot be directly compared. However, these again favor the isomer 1. To see this observe that

$$ef(Z_1 | \mathbf{1}) + ef(Z_2 | \mathbf{1}) + ef(Z_8 | \mathbf{1}) = 0.361$$
  
 $ef(Z_1 | \mathbf{2}) + ef(Z_2 | \mathbf{2}) + ef(Z_8 | \mathbf{2}) = 0.270$ 

as well as

$$ef(Z_1 + Z_8 | \mathbf{1}) = 0.043$$
  
 $ef(Z_2 + Z_8 | \mathbf{2}) = 0.020.$ 

It turns out that only the two above specified cyclic conjugation effects are responsible for the increased stability of the tribenzoperylene 1 relative to its isomer 2. These pertain to the ring 4 and to the domain spanned by the rings 1, 2, and 8.

Thus, our considerations based on energy arguments yield the same conclusion as *Clar* theory: tribenzo[*b,n,pqr*]perylene (1) is more stable than tribenzo[*b,k,pqr*]perylene (2). However, the cause of this stability difference is seen in only a few cyclic conjugation energy-effects, involving only two regions of the two molecules. Our approach distinguishes and quantifies 255 cyclic conjugation energy-effects in 1 and 2, but only two or three of these differ significantly, and only these decide the stability order.

In *Clar* theory the fact that **1** has a quintet of mutually disjoint hexagons is considered to be crucial for its stability. This, indeed, may be the case, but the collective energy-effect of these five sixmembered cycles is minor, only  $0.0025~\beta$ -units. In **2** there is no such quintet, but there are three quartets of mutually disjoint hexagons, each stabilizing **2** by some  $0.020~\beta$ -units, roughly the same as the quintet in **1**.

In conclusion, the analysis performed in this paper shows that the success of *Clar* theory cannot be explained on the basis of energy-effects caused by cyclic conjugation of  $\pi$ -electrons (and therefore remains to be a mystery of theoretical chemistry). The examples elaborated in this paper teach us that in polycyclic conjugated molecules the modes of cyclic conjugation follow a rather perplexed pattern; in an h-cyclic conjugated molecule there are  $2^h - 1$  such modes, each with a distinct (and calculable) energy-effect. However, most of these energy-effects

are small and insignificant. Only a very few energyeffects are large and may significantly differ between isomers, thus deciding their stability order.

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